DOCUMENT RESUME

ED 276 105 EA 018 929

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TITLE The Mathematization of Educational Administration: A Preliminary Analysis.
PUB DATE Apr 86
NOTE 32p.; Paper presented at the Annual Meeting of the American Educational Research Association (67th, San Francisco, CA, April 16-20, 1986).
PUB TYPE Speeches/Conference Papers (150) -- Information Analyses (070)

EDRS PRICE MF01/PC02 Plus Postage.
DESCRIPTORS Cognitive Processes; Creative Thinking; *Educational Administration; Elementary Secondary Education; Higher Education; *Mathematical Applications; *Mathematics Education; *Operations Research; Problem Solving; *Statistics

ABSTRACT

Reviews of relevant research on the application of operations research and statistics that appeared prior to 1980 are compared with those appearing in the early 1980s. In summary, both explicit and implicit users of mathematics in educational administration see it as a tool that essentially brings order and generalizability to administrative data. It is used at the stage of weighing quantitatively various alternatives that had been devised as possible responses to a problem. The last third of the paper borrows from the field of mathematics education and explores using mathematics for posing problems earlier in the problem-solving process. Suggested for posing or reposing problems are the following mathematical practices: (1) estimation/approximation; (2) internal and external views of a thing; and (3) the particular and the specific. In the field of mathematics education, mathematization takes on a different character from that derived from the physical sciences. This alternative view looks upon mathematization as the process by which something happens in an individual or a field rather than the evidence that it has happened. Examples of mathematical operations such as iteration, enumeration, relation, and transformation applied to educational administration are provided. Thinking "mathematically" is proposed as a way to encourage possible generation of more creative equivalent alternatives. Appended are 24 references. (MLF)

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The Mathematization of Educational Administration: A Preliminary Analysis

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Paper presented at the
Annual Meeting of the
American Educational Research
Association
April 1986
San Francisco

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The Mathematization of Educational Administration:
A Preliminary Analysis

During a period stretching from the late 1960s to the late 1970s applications of the branch of applied mathematics called operations research or operations analysis blossomed in the literature of educational administration. Discussants of these applications claimed that they showed great promise for reducing the amount of uncertainty and human judgment required in the administration of schools and that, coupled with increasing computer capacity, a new day of informed decision making was in view. Concurrent with this trend were increasingly sophisticated applications of another branch of applied mathematics -- statistics -- not only for the testing of hypotheses but also for the modeling of the relationships among the variables themselves (Bidwell & Kasarda, 1975).

While both of these movements remained on the fringes of the study of educational administration, with published research appearing most frequently in journals other than those in the field's mainstream, both movements were touted by those who subscribed to them (and a few others) as necessary subspecializations in the future practice of administration (Bruno & Fox, 1973; Swanson, 1974). What has happened to these attempts to mathematize educational administration in subsequent years? To what extent and in what forms have they thrived? It is the purpose of this paper to explore these questions by comparing reviews of relevant research which appeared prior to 1980 with those appearing in the early eighties. Comparison will involve the degree of mathematization as evidenced by the stated definition and essential elements, purpose, and context of the use of mathematics in the field of educational administration.

Mapping the Territory

If we take the definition of mathematization to be "the extent to which the ideas and techniques of mathematics are used (Thompson, 1981, p. 244), it is difficult to designate the boundaries of the literature to be examined for evidence. When applied to the disciplines, the application of mathematics is primarily intended for the development of understandings and insights regarding the structural relationships among relevant variables. But in a professional field such as educational administration, the potential influence of mathematics broadens to include not only the structural insights into the composition and function of the organization but also abilities to predict future states and to optimize the outcomes of the system's processes. Thus, the potential for using mathematics exists in a broad and varied set of research.

Because of the difficulty in defining the boundaries a priori, the selection of studies itself was used as a vehicle for defining the territory. Three methods of literature search were employed -- the invisible college, bibliographies, and, finally, a computerized literature search (Cooper, 1982). The first consisted of recommendations and draft manuscripts from persons known by the author to be interested in this field of research. This stretched the boundaries outside of immediate applications of mathematical models to educational planning, research, or management. Thus, material in the fields of applied mathematics, policy analysis, academic administration, organizational theory, and mathematics education was explored. Some of this literature became part of the population to be reviewed, while other pieces helped to form the context for the study of mathematization

in educational administration. Whichever these references became, they helped to set the stage for thinking about the application of mathematics to administration, rather than simply applying it.

Bibliographies of these suggested works were then scanned for relevant references. This was the least fruitful of the three methods because they tended to turn up works which were straightforward applications rather than think pieces and which were easily picked up in the computer search or works which were more theoretically rather than methodologically relevant to the source reference. Mathematization is more nearly tied to the methodological question of how one arrives at questions, hunches, or conclusions than to the actual content of the theories being tested.

Finally, a list of descriptors were derived from the analysis of these initial pieces and were used to conduct a computerized search of published research. The following sets of methodology and content terms were designed with the intention that the list of references would represent the intersection of the two sets. In other words, references selected during the search would be identified with at least one of the descriptors in the first set and the one in the second. Only published works were selected for study.

Set 1	Set 2
Methodological	Content
multiple regression analysis mathematical models operations research path analysis productivity	educational administration

References resulting from these three searches describe a crazy quilt of research in educational administration ranging from those traditionally associated with applications of mathematics in the areas of business management and planning to more mainstream studies of schools as organizations. The inclusion of the productivity descriptor in the first set was designed to include studies dealing with production function approaches to efficiency, but it actually brought many irrelevant discussions of management strategies for increasing productivity which had to be culled out of the set of studies.

This paper contains an analysis of only an initial set of literature obtained via these searches. Therefore, it should be seen as only suggestive of patterns that might be found in the entire population. A more comprehensive analysis will be carried out in a future draft.

The research reviews thus identified are analyzed here for the purpose of discovering the themes which arise surrounding the issue of mathematization. Two types of works tend to emerge -- the advocacy piece and the piece which uses mathematics rather incidentally. The former are more explicit about the advantages of using mathematics, its essential features, and where it fits into the practice and/or study of educational administration. On the other hand, authors which treat the mathematical applications as more subservient to their substantive topic tend to assume use of mathematics and to describe specific models or specific applications for the administrative tasks or decisions which form the focus of their discussion. Thus, the issues of mathematization are more implicit in the latter type of article.

Three major themes were explored in the articles from the 70s and the 80s: the definition and essential features of mathematization, the motivation for mathematization, and the context in which mathematization should take place. Each of these themes shall be described below for the purpose of observing change and/or development over time and diversity or consensus among authors. First, however, a commonly accepted tradition of mathematization will be defined and two prototype articles will be subjected to analysis so that their content might serve as a basis for comparison.

What Is Mathematization?

Thompson defines stages of mathematization based on the experience of the physical sciences (1981). In outlining those stages, he points out the immaturity of the social sciences with regard to their progress in the stages of mathematization. He suggests that there are special problems inherent in the study of humans, such as accurate measurement of certain key characteristics and the tendency of humans to resist easy conformity to laws based on measures of central tendency. Educational administration being a field which draws much of its theory from the social sciences, the prognosis for mathematization based on the physical science model would be similar. Nevertheless, the physical sciences being the only scientific model for mathematization, Thompson's stages are worthy of exploration if only to judge their appropriateness.

Briefly, he suggests that, first, a sufficient amount of data needs to exist for analysis and interpretation. Since schools are frequently charged with being data rich and analysis poor, administration would not seem to fall short in its abundance of data although its appropriateness to the problems which need to be studied and the extent to which it has

been ordered or analyzed might be questioned. The next stage involves the formulation of empirical laws or scientific principles which hold for the data. These require the discovery of patterns or regularities in the data which may be very precise as in physics or quite general as in economics. Some of the analyses of large data bases, such as High School and Beyond, (e.g., Page & Keith, 1981) qualify as examples of such formulations. Little in the mainstream educational administration literature represents pattern discovery in existing data, although some exceptions relevant to administration may be found elsewhere (e.g., Bidwell & Kasanda, 1975; Biniaminov & Glasman, 1983).

In the third stage of the physical sciences model, a mathematical structure or theoretical model which accounts for some of the laws discovered in the second stage is developed. Little seems to be said or known about how this model development takes place, although Rapoport has tried to outline this process through examples in the physical sciences (1983). While educational administration is rife with theories from which theorems are deduced and tested against real world data, these theories are seldom modeled mathematically and do not seem to account for laws which were formulated from empirical data. Thus, they do not seem to grow out of the second stage. Is this because a model of mathematization in educational administration must differ from the physical sciences or because researchers in educational administration should make more use of pattern discovery in existing data thereby conforming themselves to the physical science model?

The ultimate stage involves derivation of predictions from the model which extend into new areas of insight and may be subsequently be observed in the real world. Educational administration research is

notorious for the number of predictions derived from theory which are not borne out in observation. Whether or not this has anything to do with a failure to follow the stages of mathematization or with the inherent resistance among human actors to generalization is moot. It is clear, however, that educational administration as a field has a very spotty record with regard to the physical science model of mathematization.

But Thompson's studies seem to be limited to the development of knowledge. Rapoport broadens the potential for application of mathematics to include prediction and optimization in addition to increased insight (1983). The process of mathematization in those additional arenas remains unexplored. It is unclear whether the process of applying mathematics in those areas would be similar to or different from the physical science model.

The Prototypes

Two prototypes which represent the explicit and implicit treatments of mathematical applications are Rapoport's Mathematical Models in the Social and Behavioral Sciences (1983) from the more recent period and Mann's Policy Decision-Making in Education (1975) from the earlier period. An examination of their respective allusions to the definition, purpose and context of mathematical applications will be illustrative of the ways in which we will explore those questions in the research reviews to be analyzed in this paper.

These particular examples are selected for the light which they shed from a perspective essentially outside of the mathematical modelling literature in educational administration. Rapoport's introductory chapter contains an explicit discussion of the application of

mathematical models in the social sciences. Such explicit discussions occur occasionally in reviews of applications in educational administration (McNamara, 1972; McNamara & Chisolm, 1985) and in discussions of the place of mathematical skills in the preparation of administrators (Bruno & Fox, 1973). Comparison of discussions within our field with an example from outside should shed light on our own perspectives.

On the other hand, Mann's central theme is policy decision-making not mathematics; he seldom, if ever, even uses the term mathematics. However, his introduction of the mathematical sounding term, "calculation," in his subtitle and his use of quantification, analysis, estimating and other such terms throughout the book announces that a quantitative if not a mathematical perspective is an important part of the work. Such implicit treatments as Mann's must be included in an analysis of the mathematization of the field since the explicit research reviews of mathematical models tend to focus more narrowly on operations research applications and furthermore cannot hope to encompass the entire potential for mathematization on educational administration. Mann's book, thus, provides an example of a topically contextualized discussion of mathematical applications rather than a methodologically conceptualized one.

Defining mathematics as a language, Rapoport describes it as realizing "to the highest degree" the ideals of scientific knowledge, that is, "objectivity, generality, and precision," thereby separating both mathematics and scientific knowledge effectively from "other modes of cognition" such as those which are as "poetic, introspective, religious, tradition-preserving" (1983, p. 1). Rapoport adds that

"typically (though not exclusively) assertions in this language refer to quantities" (1983, p. 1). Characterizing the essence of mathematics as deductive, Rapoport sees its strength in abstracting only the essential features from a situation and subjecting them to the "strict intellectual discipline" (p. 25) of "rigorous deduction" (p. 18), thereby freeing the scientist from the constraints of referents in the external world. This process removes "subjective, connotative aspects of analogical thinking, leaving only demonstrable structural bases of analogy" (p. 26). Hypotheses or mathematical models generated by this method may then be used to generate predictions which may be tested against real world phenomena. Of course, by severing the model development process from real world referents the scientist achieves an uncertain picture of the external world. This trade-off is alleviated somewhat by the "quantified concept of probability" which "permits estimates of degree of certainty" relative to the real world (p. 7). Words which Rapoport uses throughout his definition of mathematics and description of its essential features include clarity, discipline, freedom, consistency, certainty, rigorous, etc. Quantification is mentioned infrequently; deduction is a central feature.

In contrast, Mann's implicit assumptions about the definition and essential features of mathematics are revealed by his use of such words as "figuring, estimating, computing" (p. 56) in his definition of calculation. While he does not use the term mathematics explicitly, he includes quantification as one of the mechanisms of calculation, which is defined as a pre-implementation component to which all functions of education "may properly be [subjected] by the superintendent" (pp. 55-6). While Rapoport's embodiment of the essence of mathematics is

deduction, Mann turns to "systems analysis as a paradigm for the calculation of organizational response" (p. 58). Thus, Mann's approach could with certainty be characterized as empirical or inductive, rather than removed from real world referents and could possibly also be considered subjective rather than objective (Beishon, 1980, p. 41). These characteristics stand in stark contrast to the essential features of Rapoport's conceptualization.

Rapoport describes the purposes of mathematization straightforwardly as prediction, optimization, and insight and organizes his discussion of models and examples around these three purposes (pp. 14-15). Mann, in contrast, implies more diffuse, even subtle, reasons for the application of calculation. He argues that systems analysis be considered a heuristic rather than an algorithm in the problem solving process. That is, since educational knowledge has not progressed to the point where we can associate specific causes and effects with certainty, there are no algorithms which can serve to provide "a guaranteed solution within a finite number of steps" (p. 83). Nevertheless, systems analysis, for example, might be a heuristic which assists the superintendent in dealing with a problem. Thus, the purpose of calculation might be to assist in any of the stages of problem solving -- recognizing demands, identifying goals, weighing alternatives, estimating probabilities, selecting actions, etc. (p. 55).

Mann, however, goes beyond the more traditional purpose of calculation and analysis and explores their utility as a political force. In fact, given the primitive attributions of causes and effects of which educational calculation is capable, he sees the policy arena as "fertile opportunity for influence" in which "control rests on, and is

partly justified by, calculation" (p. 172). Thus, however useful analysis might be as a heuristic in the problem solving process, its greatest use might be as an instrument for control.

Rapoport describes a rather broad context for mathematization, that is, the social and behavioral sciences. In categorizing the models and his examples by purpose, however, he further refines his context to situations in which prediction is desired (predictive models), situations in which optimization is desired (normative models), and a desire for insight into "the structural frameworks in which events involving human actions take place" (analytic-descriptive models) (pp. 14-15).

Mann contextualizes his discussion within the field of education and more specifically delimits his boundaries to the policy arena, that is to those issues which are broader in scope than the operational or routine but less global than those issues at the macro-societal level (p. 6). Within the policy frame, he defines a calculating (or pre-implementation) component and a control component, the latter involving implementing, enforcing, or enacting the decision (p. 55). Applications resembling mathematics, such as quantification, comparison, and making predictions, take place during the calculation component.

From a slightly different contextual perspective, both Rapoport and Mann agree that mathematics is involved only during a certain part of disciplinary or field-based processes. Mann isolates calculation from problem identification on one end of the continuum and from implementation on the other end (p. 118). Rapoport, in his focus on deduction, implicitly removes mathematization from the empirical or inductive arena. Similarly, while Thompson (1981) acknowledges that

mathematics might be involved in all the processes of his third stage, only the analysis of the mathematical structure of the model (the deductive process) is purely mathematical (p. 247).

Mathematization in Educational Administration

This initial analysis of research reviews is based on two articles from each period of mathematical applications, which are categorized by implicit and explicit treatment of mathematics in Table 1. Discussion of these four articles will be organized around the definitions and essential features, purposes, and contexts of mathematics as described by the authors. Comparisons will be made with the prototypes already analyzed in greater detail.

What is mathematics? The explicit and implicit treatments of mathematization tend to follow the general distinction noted in the prototype works in defining mathematics. Like Rapoport, both McNamara (1972) and McNamara and Chisolm (1985) use the metaphor of language to describe mathematics. Thus, in the earlier work, mathematics is defined as

> ...a branch of philosophy whose subject matter is a set of abstract entities such as sets of elements, numbers, and identified operational rules. Given this orientation, mathematics is a vocabulary of symbols that can be used to label objectives and, more importantly, a set of grammatical rules for using the vocabulary (McNamara, 1972, p. 165).

In 1985, McNamara and Chisolm quote Schwartz (1980) for a similar though more refined expression which embodies the purpose as well as the description of component parts:

> ...a grammar defining a language in which a very wide range of themes can be expressed. Thus mathematics provides a disciplined mechanism for devising frameworks into which the facts provided by empirical science will fit and within which originally disparate facts will buttress one another and rise above the primitively empirical (p. 9).

Table 1. Categories of Research Reviews

	Explicit	Implicit
Early 1965-1979	McNamara (1972)	Johnstone (1975)
Recent 1980-Present	McNamara & Chisolm (1985)	Lindsay (1982)

Implicit in the discussion surrounding both definitions is a call for a vaguely defined "mathematical literacy" among administrators. Couched in that campaign and also present in the use of the word "mechanism" in the second definition is a conceptualization of mathematics as a tool. This seems consistent with the use of the language metaphor, as language can also be conceived as a tool for representing either concrete objects or abstract ideas. The tool concept also serves to bridge the distinction between explicit and implicit discussions of mathematical applications.

Johnstone (1975) and Lindsay (1982) do not explicitly define mathematics, but seem to assume its utility as a tool. Expressions which Lindsay uses in passing include "analytical techniques" (p. 191) and "quantitative techniques" (p. 192), both of which are reminiscent of Mann's use of mechanism and quantification.

The centrality of deduction espoused so clearly by Rapoport seems to be alluded to in the Schwartz definition adopted by McNamara and Chisolm in the mathematical provision of "a disciplined mechanism for devising frameworks" (p. 9). Similarly, although less directly, McNamara's earlier article spoke of "abstraction" as an essential feature (1972, p. 165). But none of the articles honor deduction as of central import. In fact, both the Schwartz definition which speaks of "frameworks into which the facts provided by empirical science will fit" and Lindsay stress the empirical aspects of mathematical applications, suggesting perhaps the earlier, more inductive stages of Thompson's conception of mathematization.

Another persistent theme is the integrative nature of mathematical applications. Phrases such as "integration of facts" (McNamara &

Chisolm, 1985, p. 8); "synthesis" (McNamara, 1972); and consolidation of results (Lindsay, 1982, p. 192) illustrate this essential characteristic and suggest once more that the field is at the earlier stages of mathematization wherein data is being collected and patterns and laws recorded.

Quantified standards against which institutional functions may be measured is a characteristic which Lindsay points out as essential.

While each of the articles might be brief in their treatment of the definition and essential features of mathematics, they each develop a unique framework for classifying their examples. Those frameworks provide some indication of the scope of their definitions as well as the sophistication of applications at the time of the review. The earliest of our reviews, McNamara (1972), divides his administrative applications by the domains in which they may be found -- planning, theory construction, and administrative decision. Interestingly, these correspond roughly to Rapoport's prediction, analytic-descriptive, and normative models. When McNamara and Chisolm (1985) take up classification, they categorize by branches of applied mathematics -- statistics, operations research, and computer science -- rather than domains of educational administration.

Johnstone (1975) and Lindsay (1982) map narrower domains of applications consistent with their focus on more specific contexts. Classifying by methodological approaches in educational planning, Johnstone describes applications and potential applications of deterministic models, Markov Chain models, regression models, and mathematical programming models. Lindsay, on the other hand, focusing on efficiency studies, divides his analysis into studies using input-

output ratios, regression studies, and production frontier or data envelopment techniques.

Why mathematize? The purposes of mathematization as expressed by the authors of the research reviews tend to be similar to Rapoport's stated purposes. They are also, however, similar to Mann's task-oriented purposes (to weight alternatives, estimate probabilities, etc.), although they tend to be larger in scope; none of the authors cite the use of mathematical analysis as a political tool as does Mann, however.

Understanding is the most frequently identified goal of mathematical-applications, with the implication that it is factors involved in organized education that we are trying to understand (Johnstone, 1975). Lindsay (1982) states more explicitly that one of the purposes is to "improve understanding of institutional input-output relationships" (p. 191). McNamara's (1972) object of understanding differs from that of the authors of the more implicit pieces in that he argues for a better understanding of mathematics so that students of administration may understand its applications in the social sciences as well as be able to identify its usefulness and limits (pp. 171-2). In other words, he would like scholars of administration to understand the language of mathematics so that communication of basic knowledge as well as synthesis of existing information may be facilitated.

Johnstone's and Lindsay's identification of understanding as an important reason for application of mathematics is similar to Rapoport's reference to insight. Johnstone (1975) also cites prediction, another of Rapoport's reasons. While he believes understanding and prediction are not necessarily independent of each other, in practice he notes that

they generally are. That is, one may sometimes fairly accurately predict future conditions without having an understanding of the factors contributing to those conditions. He encourages more fruitful connections between the two (p. 177).

One of Lindsay's reasons also has a temporal dimension, but, interestingly, its orientation is toward the past rather than toward the future. He states that analytical techniques "provide useful information on the relative levels of performance previously achieved" (p. 191). Although he doesn't say so, perhaps this serves as an aid to understanding the current situation -- or to evaluating it.

Rapoport's normative goal for application of mathematics is probably most closely approximated by Lindsay's goal of increased efficiency for the institution in the context of declining resources and increasing public scrutiny and criticism (p. 192). Finally, an interesting reason is suggested by Johnstone in his citation of Sisson's observation that operational analysis "is the link between research and management" (p. 191). While he notes that few of the models he reviews have been developed in settings where policies are made, he holds that the potential for cooperation between researchers and practitioners is great in the model building process.

What is the context for mathematization? Next to finding a reason for the mathematical, finding a home for it has been a prominent quest for those explicitly concerned about the mathematization of educational administration. Mathematical applications are notoriously outside of the mainstream of educational administration literature and those who practice this craft are consistently seen as methodologists without a substantive or theoretical base. Their isolation and sophisticated

technical expertise has increasingly led to their being perceived as irrelevant to or out of touch with the practice of administration (Ackoff, 1979) and, in education, may be contributing to the demise of mathematical modeling.

Herbert Simon (1977) has implied that the primary connection of mathematical applications to management might lie in the area of decision making, which he defines so broadly as to encompass almost all of management. Both of the explicit articles analyzed here concur with that suggestions, the more recent one setting the context squarely and firmly in the decision sciences (McNamara & Chisolm, 1985). McNamara's earlier work describes contexts in theory construction and planning as well as decision making (1972).

The authors which deal more implicitly with mathematical applications are explicit about the aspect of administration which they treat. Johnstone (1975) and Lindsay (1982) address planning and institutional evaluation or efficiency, respectively. Perhaps the centrality of these topics in these two articles reflect the predominant issues in public thought in the two succeeding decades.

In summary, mathematics tends to be defined in educational administration by explicit expositors as a language and by those who discuss it implicitly as quantitative techniques, but both groups see it as a tool which essentially brings order and generalizability to administrative data. Its essence seems to be empirical or inductive, suggesting a focus which still resides in the early stages or mathematization, that is, the pattern finding and empirical law generating stages. In both types of writing, the purpose of mathematization is primarily to understand or to predict with a

secondary, normative purpose of changing or improving educational delivery; optimization is beyond the envisioning of these realists. The context for mathematization seems primarily to be in the decision arena -- with a seeming shift from a forward-looking planning stance in the seventies to a more retrospective and evaluative perspective in the eighties. The context of understanding the structure of administrative science is mentioned only by the author of the earlier explicit work. In short, mathematics seems to be a practical tool serving to consolidate facts around the predominant issues of the times.

Toward More Expanded Views of Mathematization

Is there an essential mathematics which is being overlooked in this paradigm of the practical? Can mathematics serve other purposes for the administrator? Is there a mathematics more inherent in administration? Can mathematical applications take us beyond the narrow issues of the era in which we dwell? Have we ever really provided sufficient justification for students of administration or public school students studying mathematics? To explore some of these queries, we will now borrow from a field in which similar questions are being addressed -- that of mathematics education.

Stretching the definition of mathematics. It was noted that each of the prototype pieces saw mathematics essentially as a tool which could be of some use primarily at the stage of weighing quantitatively the values or costs or likelihood of various alternatives which had been devised as possible responses to a problem. With the exception of McNamara and Chisolm (1985), the research reviews also implied mathematical applications which occurred long after the problem identification stage. What if mathematics is relevant to the posing of

the problem itself? Stephen Brown (1984) argues convincingly that, for a mathematician, deposing a problem is as necessary to its reposing and resolution as the derivation of a solution -- and perhaps it is a prerequisite for problem solving. Why then do we limit the relevance of mathematics to a spot very near to the end of the decision making or hypothesis testing process? Why indeed is mathematics associated only with solutions when mathematicians have demonstrated that certain mathematical problems, though unsolvable, are nevertheless of continuing interest and worth?

Brown illustrates his concern for posing (or reposing) with the presentation of the following problem:

> A fly and train are 15 km apart. The train travels towards the fly at a rate of 3 km/hr. The fly travels towards the train at a rate of 7 km/hr. After hitting the train, it heads back to its starting point. After hitting the starting point,it once more heads back toward the train until they meet. The process continues. What is the total distance this fly travels?

He continues:

> Without giving the ballgame away completely, let me suggest that an insightful and non-technical solution depends upon your asking a question that has not been asked in the problem at all. Though there are many different ways to asking the question as well as many questions to ask, something like the following will most likely be revealing:
>
> > What do I notice if I focus not on the fly as requested, but on the train instead?
>
> What is needed in the solution of this problem is s some effort at posing a new problem within the context of accepting and trying to solve a given problem. Whether or not such a problem posing is <u>always</u> needed in the solution of a problem is an interesting and debatable question (p. 17).

Let us turn to an example closer to the domain of educational administration to illustrate the potential value of deposing a problem. An exceptional amount of public and professional attention has been focused on the "problem" of class size. The problem as it is typically posed seems to be one of determining the optimal number of students which a teacher can effectively attend to and therefore instruct. Mathematical models are brought in as an aid to calculating comparisons of learning obtained in settings characterized by differing class sixes. But what if we were to approach the problem as we might productively approach the fly and train example and focus our attention not on the teacher, the seeming resource which must be "distributed" among many learners, but on the other actors in the situation, the students? Our problem might then become, How might a student best become known and therefore instructed? Such a reframing of the problem leads one to think of both children and resources in different ways. A child can be known, for example, in a number of ways -- by the ways in which he or she interacts with other children or adults, for example. Thus, other students in the class can be seen as a valuable resource instead of an added burden, and the responsibility of knowing and teaching students becomes shared rather than resting on the individual. The way is thereby cleared for alternatives which result in less pressure on individual teachers, although pupil/teacher ratios are not reduced. Family grouping, wherein four major subject area teachers share the same four classes of students among them and meet together as a team during a fifth period to coordinate efforts and share insights regarding strategies for reaching various students is one approach for the upper elementary or middle school which doesn't receive near as much attention

in the popular or professional press than the ubiquitous class size solution.

For reposing (or posing) problems, Brown outlines some interesting practices which could profitably be applied in the administrative arena. He suggests more exploration of the situation in which the problem seems embedded, i.e., dissolving an already stated problem into its component elements, as we did above or simply pondering the "things" of which a situation is composed. Those "things" might be concrete objects or beings such as the actors in our example or events like administrator contacts with students; abstractions like schedule time and space or informal social and professional networks among teachers; data like distribution of time spent by an administrator in each dyadic communication with an individual or longitudinal achievement progress made by individual students; or theorems or postulates such as the fairly well substantiated hypothesis that excellent schools tend to be overseen by administrators who are outstanding leaders. The categories of "things" which might be the elements of situations could undoubtedly be expanded well beyond this suggestive list and would probably vary from situation to situation. With these four in mind, however, we can turn to some of Brown's suggestions for posing or reposing problems.

(i) Estimation/approximation

We spend a great deal of effort wringing our hands over the inaccuracy of the data we are able to collect in attempt to measure certain abstract concepts and then we turn and force that "vague" data into sophisticated statistical models or firm measures of the academic well-being of a school district. Perhaps as much value could be gained by softer descriptions of the approximate educational experiences of

students and rough estimates of faculty and student achievement in a variety of areas beyond the simply intellectual.

(ii) Internal and external views of a thing

A recent piece of research (McDermott, Goldman, & Varenne, 1984) viewed school assigned homework from the perspective of the home and achieved some significant insights for school practice. The opportunities for climbing inside or outside of a typical administrative perspective are legion and advantages of doing so for problem reposing seem promising. To take one of our examples of an abstract "thing" suggested above, schedule time and space, do any computerized scheduling software packages allow the administrator to simulate the experience of a student in order to identify the ease or stress which the students experiences in conforming to the schedule which has been prescribed by a concern with efficiency?

(iii) The particular and the specific

In our search for and use of index numbers which serve as indicators of institutional achievement and research which explores generalizable causes and effects, we seem to have lost sight of the possibility of solutions which are situation or person-specific. Ebel's (1982) suggestion that the situation specific aspect of operations research might make it an appropriate model for educational research was a "voice crying in the wilderness" which has remained largely unheeded. The indicators of neglect of the large majority of "ordinary" students uncovered in studies like The Shopping Mall High School serve notice to the field that particular students in specific situations are in need of administrative and programmatic attention. What can be learned from rich, in depth data bases like High School and Beyond and the sustaining effects study of compensatory education (Carter, 1984) beside the

generalized models which have been derived from them?

A second approach to problem generation which Brown proposes, in addition to this resolution of problems into situations and exploration of those situations, is one which he likens to adolescent rebellion. It is embodied in the question, "What if not?" To take an example from the academic setting, it is often deemed desirable for newly enrolled students to be exposed to a variety of perspectives for the purpose of giving them exposure to either a broad range of knowledge or a sample of faculty. Implicit in this design of introductory programs is a valuing of student synthesis of knowledge prior to student choice among perspectives. But what if synthesis is not seen as the domain of the student? What kinds of alternative problem situations might be unveiled?

> Why not focus on faculty involvement with one another, so that each of them are in a strong position to provide incoming students with the variety and interrelatedness of the field?
>
> Is a broad range of exposure necessary or sufficient for choice? What would happen if students began by choosing?
>
> What if, at the doctoral level, students are responsible for their learning? What does faculty responsibility become?

Such technologies which we have explored above may seem closer to critical and creative thinking than to the domain of mathematics. But mathematics educators (Burton, 1984; Wheeler, 1982) specifically link these processes to what mathematicians do, or to "mathematical thinking." That it is not developed in traditional instruction in high school and college mathematics or in operations research courses is a source of common concern to both mathematics educators and operations research scholars (Burton, 1984; Ackoff, 1979a, 1979b). It also is quite likely to be the source of the limited use of techniques of applied mathematics which we have reviewed in our analysis of mathematization.

How does one think mathematically?

In the field of mathematics education mathematization takes on a different character than the one derived from the physical sciences. This alternative view looks upon mathematization as what a mathematician does; it is the process by which something happens in an individual or a field rather than the evidence that it has happened. In studying this concept of mathematization, scholars would ask questions like

> What causes individuals to collect data or to pay attention to data that is collected?
>
> What does one do with the data that makes it meaningful?
>
> What are the mental processes involved in exploring, reframing, or resolving problems?

Burton (1984) has outlined a model of mathematical thinking which is an attempt to describe the operations, processes, and dynamics involved.

Identifiably mathematical operations such as iteration, enumeration, relation, and transformation may be performed on any of the things or elements which were suggested in the previous section. Iteration, or simple repetition, of the elements of a set of objects is mathematical in that it assumes recognition and continuation of a pattern. Similarly, cardinal or ordinal enumeration is a mathematical operation which takes place whenever, for example, an administrator has to respond to the question, How many?

The mathematical operation of relating occurs in a number of specific ways. For example, when the schools in a district are listed by age, size or average achievement test score, they are being related by ordering. When correspondences between such a list and a similar list in another school district or a list of the same schools ordered by another variable, such as proportion of students receiving free lunch, another form of relating is occurring. When schools with a large proportion of students from low income families receive large amounts of Chapter I funding and are provided with more support personnel such as social workers, aids, and security guards than schools with a medium or low proportion of students from low income families, equivance classes are formed. Finally, when it is noted that the higher the proportion of students from low income families in a school, the lower the average achievement of the school, an inverse relationship is being described.

Great transformation took place and is continuing to occur by the combination of small rural districts into consolidated units. And, in urban settings, policy-makers have attempted to transform schools by substituting groups of students with different characteristics for portions of the existing student body through desegregation and/or

magnet school programs. Thinking of these operations as mathematical at least helps in the planning of their execution, but also may encourage more perceptivity with regard to their potential impacts and the possible generation of more creative equivalent alternatives.

Burton's processes of mathematical thinking consist of specializing, conjecturing, generalizing, and convincing. In that order, they can be associated with inductive reason, whereas reversing the order of the first three provides us with a model of deductive thinking. The element of convincing which concludes each form of process is the step by which one moves beyond the personal to the public, a notion very familiar to the administrator. Involved in this part of the process is the concept of the monitor, or internal opponent, which raises doubts, questions assumptions, and generally aids the individual in developing a defensible argument. The result of such a process is analogous to the mathematical proof.

Burton's dynamics of mathematical thinking are represented by helical model wherein manipulating, getting a sense of pattern, and articulating the pattern symbolically repeat in continuous spirals of sophistication or diversity. Such dynamics, I suspect, would be intimately familiar to the administrator who is continuously trying to make sense of the educational environment of which he or she is a part.

These views of the application of mathematics which come out of the field of mathematics education suggest an involvement which may be inherent in the discussions in educational administration but are certainly not made explicit there. That is, to return to the metaphor of language, the idea of mathematical literacy goes well beyond the mere learning of a vocabulary and its corresponding grammatical rules.

Literacy to the mathematics educators requires an involvement with the language, a facility in its use, a conscious or self-conscious awareness of one's own thought processes and mental manipulations.

But this concept of mathematization also propose a message which is radically different from, even opposed to, that of the operations research or mathematical modeling proponents. The latter's valuing of consistency, generalizability, certainty, and objectivity as essential features of mathematics is enriched by a relishing of inconsistencies, diversity, uncertainty, subjectivity. One of the barriers to the application of mathematics in the human sciences which is noted by Thompson is that humans don't seem to manifest the regularity found in the physical sciences, and thus the generalizations possible in mathematics may not be found to model human behavior. Perhaps a more open and radical view of mathematics is precisely (or imprecisely?) what would be applicable.

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